

Bursts from the very early universe

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Bursts of weakly interacting particles such as neutrinos or even more weakly interacting particles such as wimps and gravitons from the very early universe would offer a much deeper “look back time” to early epochs than is possible with photons. We consider some of the issues related to the existence of such bursts and their detectability. Characterizing the burst rate by a probability \mathcal{P} per Hubble four-volume we find, for events in the radiation-dominated era, that the natural unit of description is the present intensity of the CMB times \mathcal{P} . The existence of such bursts would make the observation of phenomena associated with very early times in cosmology at least conceptually possible. One might even hope to probe the transplanckian epoch if complexes more weakly interacting than the graviton can exist. Other conceivable applications include the potential detectability of the formation of “pocket” universes” in a multiverse.

I. INTRODUCTION

The thought of observing bursts of neutrinos or even more weakly interacting particles from cosmologically early times raises a number of interesting issues [1]. While most of the history of the very early universe is conventionally described as a succession of thermal quasi-equilibrium states, in the more recent universe violent phenomena such as neutrino bursts, gamma-ray bursts, or cosmic ray flares frequently occur. It is perhaps not excluded that bursts of various kinds also exist from the earlier universe. Particularly if these bursts were to contain weakly interacting particles, the particles would be “messengers” carrying information from very early epochs. Time-of-flight effects, for example, can provide the opportunity to determine the parameters of the spacetime geometry at the early epoch.

Ignoring for the moment the difficult problem of detectability, this nevertheless raises the intriguing question of what is potentially observable from earlier and earliest epochs, and in this note we would like to discuss the in-principle direct observability of such very early, non-thermal phenomena. This is at least of philosophical interest in connection with ideas like the “landscape” or “multiverse” inspired by string/M theory, or eternal inflation and quantum gravity implications of black hole formation and “baby universes” [2]. While these events often lead to new regions of causally disconnected space, it does not seem implausible that they would leave some trace on our past light cone. The possibility of direct observation of such events –even in principle– moves such ideas out of the realm of purely non-physical speculation into the world of at least “gedanken experiments”.

In general, the more weakly interacting the particle, the further back a burst of some kind can reach us without being lost through thermalization by the intervening matter and radiation. While the “look back time” with photons is limited to a few hundred thousand years after the big bang, with neutrinos we can get to a few seconds (ν_e) or milliseconds (ν_μ, ν_τ), with the yet to be discovered wimps to perhaps picoseconds, and with gravitons to the epoch of inflation or to the Planck time. Because of the general relativistic effects involved with such early times or high redshifts z , a number of interesting features would be involved with these phenomena, which we would like to briefly identify.

II. FLUX DILUTION, DURATION, AND ANGULAR SIZE

We discuss some features that might be expected for bursts emitted at cosmic time t_{em} as they arrive at the earth. We use the simplest schematic FRW cosmology; the necessary quantities are briefly reviewed in the appendix. The spatial curvature is taken to be zero. We concentrate on the case of zero mass or highly relativistic particles.

Flux Dilution: The intensity of a burst upon reaching the earth will of course be greatly reduced by a geometric flux dilution factor. However, this factor rapidly approaches a limit at early times. In the flat space result, that for a source emitting N particles the number traversing unit area at a distance D is $N/(4\pi D^2)$, we use the D of Eq 31 (Appendix), so that

$$\text{no. crossing unit area} = N \frac{1}{4\pi D^2} \approx N \frac{1}{4\pi} \left(\frac{1}{3t_{now}} \right)^2 \approx N \cdot 6 \times 10^{-59} / \text{cm}^2 \quad z \gg 1. \quad (1)$$

The limiting value for D is essentially reached by $z \sim 10$ and thereafter there is no significant further reduction as z increases. Thus for these early times, the number of particles potentially available for detection depends only on N , the number emitted. We assume isotropic emission; Eq 1 can be enhanced if there are beaming effects, as with the jets of galaxies, where the emission is into less than the full 4π of solid angle.

Angular size: D_H is the size of a causally connected region at t_{em} , while the dimension of our backward light cone is $R(bl)$. Since the burst can at most come from a region of size D_H , the angular dispersion of the burst is limited by $D_H/R(bl)$

$$(angular\ dispersion) \approx \frac{(t_{em}/t_{now})^{1/3}}{1 - (t_{em}/t_{now})^{1/3}} = 1 \times 10^{-6} \frac{(t_{em}/s)^{1/3}}{1 - (1 \times 10^{-6})(t_{em}/s)^{1/3}} \text{ rad} \quad t_{em} > t_{eq} \quad (2)$$

$$(angular\ dispersion) \approx \frac{2}{3} \frac{(t_{em} t_{rad})^{1/2}}{t_{now}} = 7 \times 10^{-9} (t_{em}/s)^{1/2} \text{ rad} \quad t_{em} < t_{eq} \quad (3)$$

Thus for not too early times, there is a conceivably detectable angular spread of the burst, if it is emitted from a horizon-sized region.

Burst duration : The redshift stretches all time scales. Thus a neutrino burst-producing event at the neutrino decoupling time of $\sim 1s$, if it lasted one ms then, would last a year upon reaching us today [1]. It would then seem that even potentially large bursts would be stretched to invisibility. However, there is a consideration working the opposite way. Burst events, representing a causally connected phenomenon, cannot last longer than D_H or the age of the universe at the time they occur, $\sim t_{em}$. This argument is similar to the one used in inferring the maximum size of a source from flaring phenomena. Thus we would conclude that the maximum duration τ_{now} of a burst upon reaching us is $\sim t_{em}/a_{em}$ and so for early times

$$\tau_{now} = 2t_{em}/a_{em} = 2(t_{rad}t_{em})^{1/2} = 9 \times 10^9 (t_{em}/s)^{1/2} \text{ s} \quad t < t_{eq} \quad (4)$$

Thus the decreasing time scale wins out over the redshift and the time scale for burst shortens at early times. At later times, of course structures involving time scales shorter than the Hubble time can form. Thus we would expect, as we go further back, that the bursts first tend to lengthen at moderate z 's, and then tend to get shorter, following Eq 4. At the QCD phase transition, for example, Eq 4 gives a burst duration of about a year, while at the Planck time it is $\sim 10^{-12}s$.

III. FLUXES

A prime question, evidently, for the conceivable observability of the bursts is the intensity or rate to be anticipated. We proceed on the assumption that at a given epoch of cosmic time there is a uniform population of objects which can emit a certain type of burst with some probability during an interval dt_{em} around t_{em} . Let this emission probability be described by $\rho(t_{em})$, a rate per cm^3s for the burst rate at the epoch t_{em} . With E_{burst} the energy of the burst in the local rest frame at that epoch, bursts in the time interval dt_{em} will create a uniform radiation field with an energy density

$$d(\text{energy density})_{em} = E_{burst}\rho dt_{em} \quad (5)$$

Redshifted to the present with the factor a_{em}^4 appropriate to an energy density of relativistic particles this becomes

$$d(\text{energy density})_{now} = a_{em}^4 E_{burst}\rho dt_{em} \quad (6)$$

for the present energy density of this radiation. For relativistic particles, this uniform energy density implies an energy flux per cm^2s at a detector

$$d(\text{energy flux}) = \frac{1}{3} d(\text{energy density}) = \frac{1}{3} a_{em}^4 E_{burst}\rho dt_{em}. \quad (7)$$

The quantity ρ represents the rate of bursts per unit four- volume, and it seems most natural to express it in terms of the characteristic dimensions at t_{em} . From Eq 28(Appendix), the spatial density of horizon sized volumes at early

times is $(\frac{4\pi}{3}(2t_{em})^3)^{-1}$. Let the probability per unit time that one such volume produces a burst be proportional to some probability factor \mathcal{P} in some time interval. The only natural quantity for the time unit is again the Hubble time $1/t_{em}$ so we write

$$\rho = \frac{1}{\frac{4\pi}{3}(2t_{em})^4} \mathcal{P} \quad (8)$$

for the density of bursts per unit spacetime volume. \mathcal{P} is a dimensionless, in general t_{em} -dependent quantity, and presumably small. In writing this, Eq 7 becomes

$$d(\text{energy flux})_{now} = \frac{1}{4\pi} a_{em}^4 E_{burst} \frac{1}{(2t_{em})^4} \mathcal{P} dt_{em}, \quad (9)$$

or, using $a = (t/t_{rad})^{1/2}$ for early times and Eq 27 (Appendix)

$$d(\text{energy flux})_{now} = \frac{\pi^2}{90} E_{burst} \frac{T_{now}^4}{(M_{pl} t_{em})^2} \mathcal{P} dt_{em}, \quad (10)$$

Size of bursts: The question now arises as to how large E_{burst} might be. It seems difficult to give a complete answer to this question without a theory of quantum gravity, particularly for the gravitational radiation accompanying abrupt changes in spacetime. However the only obvious quantity and presumably that which corresponds to the maximal burst is the energy within the horizon at t_{em} . In such a “horizon collapse” we would assume that all the energy within the causal horizon is radiated away, presumably in the form of particles with energies higher than that corresponding to the original thermal plasma. Again, an estimate of this new energy scale appears to involve quantum effects, and one simple suggestion would be that it is M_{pl} .

These “collapses” might be possible with inhomogeneous universes where there initially were occasional but large density variations. In this case the bursts might be the sole evidence of the “failed universes”. On the other hand if there was an inflation and the universe is highly homogenous, such events would be unlikely. However they might arise in phenomena of the “baby universe” or the “universe in the lab” type where an horizon-sized region undergoes an inflation and some part of the event remains in causal contact with us. Evidently we must assume that these events are random and rare in order to continue to use simple isotropic-homogeneous cosmology.

Adopting this energy within the horizon as an estimate, we set $E_{burst} = E_{horizon}$ in Eq 10, where from Eq 29 we have $E_{horizon} = M_{pl}^2 t_{em}$ and so obtain the rather simple formula

$$d(\text{energy flux})_{now} = \frac{\pi^2}{90} T_{now}^4 \mathcal{P} \left(\frac{dt_{em}}{t_{em}} \right) \quad (11)$$

for the energy flux at the present from the epoch t_{em} .

The resemblance of this result to the intensity of the CMB can be understood as follows. The normalization in Eq 8 to the horizon volume states that the burst rate is $\mathcal{P} dt_{em}/(2t_{em})$ per horizon volume. If we then assume the energy of the burst is that in a horizon volume or $u_{em} \times \text{horizon volume}$, the horizon volume cancels out, leaving the energy density $u_{em} \mathcal{P} dt_{em}/(2t_{em})$. The evolution of the energy density u_{em} to the present gives the present CMB, and so we obtain

$$d(\text{energy flux})_{now} = d(\text{energy flux})_{cmb} \mathcal{P} \left(\frac{dt_{em}}{2t_{em}} \right), \quad (12)$$

which is the same as Eq 11.

For constant \mathcal{P} the formula gives an energy flux coming about equally from all epochs; as we go to earlier times, the decreasing $E_{horizon}$ is compensated by the increasing number of emitting regions. The only dimensional constant explicitly appearing is T_{now} , the present temperature of the microwave background, or alternatively the energy or time scale in the radiation dominated epoch via Eq 26 or Eq 27. However, various constants will presumably be involved in \mathcal{P} .

Energy per burst: As for the energy in an individual, *single* burst, Eq 29 (Appendix) red-shifted to the present (with relativistic particles) and with the flux dilution factor from Eq 1 gives the energy crossing per cm^2 as

$$E_{horizon}^{now}/\text{cm}^2 = E_{horizon} \frac{a_{em}}{4\pi D^2} = \frac{M_{pl}^2 t_{rad}}{4\pi (3t_{now})^2} (t_{em}/t_{rad})^{3/2} = 3 \times 10^3 (t_{em}/s)^{3/2} \text{ eV}/\text{cm}^2, \quad (13)$$

for the energy crossing unit area at the detector, from a single horizon-sized burst, from $t_{em} < t_{eq}$.

Thus an individual, originally horizon-sized burst from around $t_{em} \approx 0.01s$ would appear in a cm-sized detector as simply a small- eV sized – but athermal signal. “Athermal” signifies that the energy or effective temperature is larger than that of the relic distribution of the particle in question. In addition, the “burst” is spread over a considerable time period, according to Eq 4.

Burst Rate: In addition to the energy flux we can also note the burst *rate* corresponding to an assumed $\rho(t_{em})$. The number of events from an interval of cosmic time dt_{em} and arriving to us during an interval Δt will be $\rho dt_{em} \times (surface\ area) \times \Delta R$, where ΔR is the separation of distance at the epoch t_{em} leading to an arrival time difference Δt . The relation is $\Delta R = a_{em}\Delta t$. Thus since on our backward light cone $(surface\ area) = 4\pi R^2$ with R given by Eq 33, the rate is

$$d(bursts/s) = \rho\ 4\pi R^2 a_{em} dt_{em}, \quad (14)$$

We note that this burst rate, when multiplied by the energy per burst, Eq 13, and using $R = a_{em}D$ yields Eq 6 for the energy density, checking the consistency of the formulas. Inserting Eq 8 for ρ

$$d(bursts/s) = \frac{3}{16} D^2 \mathcal{P} \left(\frac{1}{t_{em} t_{rad}} \right)^{3/2} \frac{dt_{em}}{t_{em}} = 5 \times 10^6 \mathcal{P} \frac{1}{(t_{em}/s)^{3/2}} \frac{dt_{em}}{t_{em}} /s. \quad (15)$$

Thus if the intrinsic probability at $t_{em} = 1s$ were $\mathcal{P} = 10^{-6}$, there would be about one burst per second originating from this epoch. Originating from the Planck time there would be $\sim 10^{65}$ bursts per second with this \mathcal{P} . The burst rate is more strongly divergent at small t_{em} than the energy flux since it is not weighted with the decreasing energy per burst.

“Olber’s paradox”: We may wonder if with the very rapidly increasing number of independent emitting regions we do not finally produce a kind of “Olber’s paradox”, where the amount of energy received from earliest times diverges. Given an assumption on the t_{em} behavior of $\mathcal{P}(t_{em})$, the total energy received can be found from Eq 11

$$(energy\ flux)_{now} \propto \int \mathcal{P} \frac{dt_{em}}{t_{em}}. \quad (16)$$

Thus if \mathcal{P} is constant there is a mild Olbers Paradox in the form of a logarithmic divergence. With a cutoff presumably at the Planck time this would give a further factor $\ln(M_{pl} t_{now}) \sim 10^2$ in the flux estimate $CMB \times \mathcal{P}$. The magnitude and form of $\mathcal{P}(t_{em})$ is of course the great unknown in our estimates.

Recent epochs: The estimates in this section were all for t_{em} in the radiation dominated epoch. While horizon-sized bursts do not seem very likely after t_{eq} , where the horizon is already tens of thousands of years, for completeness we give the analog of Eq 11:

$$d (energy\ flux)_{now} = \frac{16}{81} \rho_{now} a_{em} \mathcal{P} \frac{dt_{em}}{t_{em}} \quad t_{em} > t_{eq} \quad (17)$$

which again follows from using the Friedman equation, with ρ_{now} the present matter density.

IV. “OPTICAL DEPTHS”

We have, by assumption, taken the “medium” to be transparent to the particles of the burst. However as go back to earlier and earlier times the increasing density of the universe will lead to a greatly increasing column density and limit the “optical depth” from which a particular particle will be able to reach us. To examine the “transparency” we attempt some rough estimates.

“Energy Depth”: A burst from very early times will travel through rapidly varying epochs with a high but decreasing matter and radiation density. The effects of this will of course depend very much on the nature of the interaction of the burst and this medium, but for general orientation it is perhaps useful to estimate the “column density” traversed by a burst. Without any specific assumptions as to the nature of the particles and their interaction, the only plausible measure would seem to be the energy/cm² along the “line of sight”. Calling this quantity d , we integrate the energy density along the “line of sight”.

$$\begin{aligned} d &= \int_{t_{em}}^{t_{now}} u dt \approx \frac{\pi^2}{15} T_{now}^4 (t_{rad})^2 \int_{t_{em}}^{\infty} (1/t)^2 dt = 4 \times 10^{48} \frac{1}{(t_{em}/s)} \text{ eV/cm}^2 \\ &= 7 \times 10^{15} \frac{1}{(t_{em}/s)} \text{ gm/cm}^2 \quad t_{em} < t_{eq}. \end{aligned} \quad (18)$$

At the Planck time this reaches the rather substantial value of $\sim 10^{58} \text{ gm/cm}^2$.

Gravitons: In terms of specific particles, we can consider the optical depth for the most weakly interacting particle we presume to know about, the graviton. To do this, we need the cross section for the interaction of a graviton with the background medium, which just on dimensional grounds would be

$$\sigma \sim G^2 E \omega, \quad (19)$$

with E the energy of the graviton and ω the energy of the background particle. The mean-free-path for the graviton is then $\lambda = 1/(\rho\sigma)$, with ρ the number density. Thus $\omega\rho$ or the energy density u arises, and

$$1/\lambda \sim G^2 E u. \quad (20)$$

To estimate from what “depth” a graviton would reach us we should find that t_{em} for which

$$\int_{t_{em}}^{t_{now}} (1/\lambda) dt \approx 1 \quad (21)$$

If we take a graviton emitted at the Planck time with energy M_{pl} there is actually no need to explicitly do the integral. It will be seen that (with our usual neglect of the number of degrees of freedom factor) u and all other factors involve only M_{pl} . Since there is no other constant involved for the dimensionless integral Eq 21, we will obtain $\int_{t_{em}}^{t_{now}} (1/\lambda) dt \approx \int_{t_{pl}}^{\infty} (1/\lambda) dt \sim 1$. Hence we conclude that an $E \sim M_{pl}$ graviton emitted at the Planck time can just about reach us. Lower energy or more recently emitted particles will naturally find it easier to reach us. On the other hand more strongly interacting or perhaps higher energy transplanckian single particles will be absorbed, if their interaction is as in Eq 19.

In general, since the burst particles are presumed to be emitted with energies at or above the temperature of the ambient medium, they should travel freely at times around or somewhat later than that for the decoupling of that species. Thus taking this time, for neutrinos we anticipate $t_{em} > 10^{-3} s$ and for a wimp χ , $t_{em} > 10^{-11} (m_\chi/100 \text{ GeV})^{-3/2} s$, while with gravitons we can go back to $t_{pl} = 0.6 \times 10^{-43} s$.

Recent times: It is interesting to note that the d of Eq 18 reaches about a thousand grams, the depth of the earth’s atmosphere, around $t_{em} \approx t_{eq} = 2 \times 10^{12} s$. This suggest that also the usual components of cosmic rays, protons and photons, can reach us from epochs before the present one. More specifically, now working with the baryon component and taking the baryon density at 0.04 of the critical density, we have at present an average baryon density of $\rho_{now} = 4 \times 10^{-31} \text{ gm/cm}^3$. Scaling with a , Eq 23 (Appendix), we have a column density of baryons for a particle emitted at $t_{em} > t_{eq}$

$$d_{baryon} \approx 4 \times 10^{-31} \text{ gm/cm}^3 \int_{t_{now}}^{t_{em}} (1/a^3) dt = 2 \times 10^{15} \left(\frac{1}{(t_{em}/s)} - \frac{1}{(t_{now}/s)} \right) \text{ gm/cm}^2, \quad (22)$$

which yields a thousand grams at $t_{em} \approx t_{eq}$. Thus matter-radiation equality is around the “horizon” for ordinary cosmic rays and sets the limit where weakly interacting particles become necessary for a burst to reach us [6].

Transplanckian epoch: At the opposite end of the time scale, we may speculate concerning the existence of direct signals from the quantum gravity or transplanckian epoch. This leads us to a curious question: is it possible to have an arbitrarily weakly interacting particle?

The question arises since according to the arguments around Eq 21 it might appear that we need only postulate the existence of a particle whose interactions are weaker than that of the graviton, less than Eq 19, in order for the particle to reach us from “before” the Planck time. On second thoughts, however, this is not possible within the framework of conventional particle physics. Gravity couples universally to all forms of energy, and so any particle with mass or energy must inevitably have at least an interaction on the order of $\sigma \sim G^2 E_{cm}^2$. Thus it would appear we are forced to conclude that transplanckian signals cannot reach us.

A way around this seemingly unavoidable conclusion, suggested to us by V. I. Zakharov, might be offered by string theory or other models where the graviton is imbedded in a complex representing many particles, like the “string”. The whole string or complex could be more weakly interacting than the graviton itself. In this way the object might be able to “get out”, even though viewed as individual components this is not possible. This is an interesting line of thought, but goes beyond the straightforward particle - based ideas we use here.

V. CHARACTERISTICS OF THE SIGNAL

We discuss some aspects of the signal following from the above considerations. In doing so we are not, unfortunately, able to deal with the most important point, the detection method. This promises to be very difficult since by

assumption we deal with very weakly interacting particles. Nevertheless some aspects of the presumed signal are novel and interesting.

For not too early times, the burst will qualitatively be like those familiar from, say, supernova neutrino bursts or gamma-ray bursts. As we go to earlier times, and if \mathcal{P} is not extremely small, we likely have the situation of a high rate of low energy bursts—a steady “rain” of weak signals. Since the particles are by assumption originally more energetic than that of the ambient plasma at the time of emission, the individual quanta should be more energetic than those of the present CMB and so not of extremely low energy. Thus, unless the detector is of very large area or volume, the burst appears essentially as isolated single quanta. On the other hand, the fact that we indeed deal with a burst and not a random signal offers the possibility of coincidences between separated detectors, should a sufficiently efficient detection method exist.

There are thus two qualitatively different situations according to whether the burst rate or equivalently the time between bursts is large or small compared to the length of the bursts. If the bursts arrive singly there is the possibility in principle of extracting some kind of coincidence or coherence information, perhaps from multiple detectors. On the other hand if many bursts overlap in time this is not possible; unless, that is, the detectors have good angular resolution and can identify individual bursts by direction.

Finally, we note that a most interesting question concerning the signals would be directional isotropy — or the lack of it. The directional isotropy is one of the most striking feature of the CMB, and it would be of great interest to know if this is still the case for radiation from earlier epochs.

VI. DEGREES OF FREEDOM, INFLATION, DARK ENERGY

We have made a number of simplifying assumptions, including considering only one radiation field ($g(T) = 2$) [3], and the neglect of inflation and dark energy, if they exist. Concerning dark energy or a late-time expansion, its effects are anticipated to only become relevant in recent epochs where, as for the observations of type 1a supernovae, they are small and should not have significant effects on the above considerations. Concerning $g(T)$, the entropy conservation condition for the interacting plasma $(Ta)^3 = \text{constant}$ or $T \sim 1/a$ becomes $T \sim 1/(g^{1/3}a)$ [4], so that instead of $u \sim 1/a^4$ for the energy density one has $u \sim 1/(g^{1/3}a)^4$. If one makes an adiabatic assumption so that g may be treated as a constant in the various equations, g appears in the form of small fractional powers and our general estimates will not be qualitatively altered. Interestingly the effects cancel in Eq 11, which remains unchanged.

If we adopt the inflationary scenario, the previous considerations can still be used, with the proviso that they be used after the end of inflation. Any bursts emitted before inflation will of course be inflated away, shielding the quantum gravity epoch from view. The associated smoothing of the energy density, leaving only the $\sim 10^{-5}$ fluctuations later seen at recombination, would tend to eliminate potentially more violent fluctuations. It thus appears that the absence of very early bursts is a prediction of inflation. On the other hand the possibility of inflationary phenomena like “baby universes” might still leave burst-like traces in our history. Although a more detailed study of such occurrences would be interesting, qualitatively the scale for such phenomena would appear to be again the horizon, as in our discussion.

VII. SUMMARY

We have suggested that, in principle, it would be possible to detect bursts of weakly interacting particles such as neutrinos or even more weakly interacting particles such as wimps and gravitons from the very early universe. This would offer a much deeper “look back time” to earlier epochs than is possible with photons. We have considered some of the issues related to the existence of such bursts and their phenomenology. The existence of such bursts would make the observation of phenomena associated with very early times in cosmology, or perhaps even the transplanckian epoch, at least conceptually possible. For the latter it seems necessary to posit the existence of complexes more weakly interacting than the graviton. Possible applications include the conceivable detectability of the formation of “pocket” universes” in a multiverse as well as similar phenomena. In this application we should stress that we do not wish to suggest in any way that there can be contact or information transferred from “other universes”. Rather the bursts we discuss would be edge effects or transition phenomena taking place during the short periods of formation of the new region of spacetime.

APPENDIX: Description of the early universe

We briefly review the basic quantities used in our simple cosmological model (units $\hbar, c, k_{Boltzmann} = 1$):

Scale Factor: We use the scale factor $a(t)$, which at present is $a_{now} = 1$. For times after matter- radiation equality at t_{eq} , one has $a \sim t^{2/3}$ and before t_{eq} , in the radiation dominated epoch, $a \sim t^{1/2}$

$$a = (t/t_{now})^{2/3} \quad t > t_{eq}; \quad a = (t/t_{eq})^{1/2} \quad a_{eq} = (t/t_{rad})^{1/2} \quad t < t_{eq}; \quad (23)$$

where we introduce the quantity $t_{rad} = t_{eq}/a_{eq}^2$. We thus have the parameters

$$t_{eq} = 2 \times 10^{12} s \quad t_{now} = 4 \times 10^{17} s \quad t_{rad} = 2 \times 10^{19} s \quad a_{eq} = 3 \times 10^{-4}. \quad (24)$$

We use t_{em} to refer to the time of emission of the burst. The redshift of an energy or frequency from t_{em} to the present is $E_{now}/E_{em} = a_{em} = 1/(1+z)$ or $\approx 1/z$ for large z .

Energetics: Also, we need the energetic quantities energy density, temperature and the energy in a horizon volume at early times. We work with a simplified universe with only one radiation field, so that also for the temperature

$$T_{now}/T(t) = a(t), \quad (25)$$

where T is the temperature of the radiation field and in particular T_{now} is the present temperature of the CMB.

For the radiation-dominated era we thus have for the energy density

$$u = \frac{\pi^2}{15} (T_{now}/a)^4 \quad t < t_{eq}. \quad (26)$$

With more fields present at high temperature, the effective T_{now} in Eq 26 will change somewhat, however for our present semi-quantitative purposes we shall take it to be the constant $T_{now} \approx 3^0 K \approx 2.5 \times 10^{-4} eV$.

We note that the Friedman equation in the radiation dominated era connects T_{now} from Eq 26 and t_{rad} in Eq 23:

$$T_{now}^2 t_{rad} = \sqrt{\frac{45}{32\pi^3}} M_{pl}. \quad (27)$$

Distances: To find the energy within the horizon at some early time t_{em} , we first need the size of the horizon, which is given by $D_H = a_{em} \int_0^{t_{em}} (1/a) dt$, leading to [3]

$$D_H = 2t_{em} \quad t_{em} < t_{eq}; \quad D_H = 3t_{em} \quad t_{em} > t_{eq}. \quad (28)$$

The energy within the horizon at early times now follows by multiplying u in the Friedman equation by the volume factor $\sim D_H^3$ to obtain

$$E_{horizon} = u \frac{4\pi}{3} (2t_{em})^3 = M_{pl}^2 t_{em} = M_{pl} (t_{em}/t_{pl}) \quad t_{em} < t_{eq} \quad (29)$$

using $a \sim t^{1/2}$ and the Planck time $t_{pl} = 1/M_{pl} = 0.55 \times 10^{-43} s$. For later times the same argument leads to essentially the same result

$$E_{horizon} = 6M_{pl}^2 t_{em} \quad t_{em} > t_{eq}. \quad (30)$$

We also need the distance $D = \int_{t_{em}}^{t_{now}} (1/a) dt$, which is given essentially by the matter dominated epoch:

$$D \approx \int_{t_{em}}^{t_{now}} (1/a) dt = 3t_{now} [1 - (t_{em}/t_{now})^{1/3}] \approx 3t_{now}. \quad (31)$$

D enters into the radius of our backward light cone, $R(blc)$ at t_{em} via $R(blc) = a(t_{em}) \int_{t_{em}}^{t_{now}} (1/a) dt$. so that

$$R(blc) = a_{em} D = 3t_{now} [(t_{em}/t_{now})^{2/3} - (t_{em}/t_{now})] = 3t_{em} [(t_{now}/t_{em})^{1/3} - 1] \quad t_{em} > t_{eq} \quad (32)$$

and at early times

$$R(blc) = a_{em} D = 3t_{now} (t_{em}/t_{rad})^{1/2} \quad t_{em} < t_{eq}. \quad (33)$$

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- [1] L. Stodolsky, Physics Letters **B 473**, 612000, and Looking Back with Neutrinos, *Proceedings, Carolina Symposium on Neutrino Physics*, Columbia, March 2000; Eds. J. Bahcall, W. Haxton, K. Kubodera, and C. Poole; World Scientific, Singapore. astro-ph/0006384.
 - [2] Testability issues connected with such ideas have recently been reviewed by P. Davies, astro-ph/0602420, G. Ellis astro-ph/0602280, or W. Stoeger astro-ph/0602356.
 - [3] E. Kolb and M. Turner, *The Early Universe*, Addison-Wesley, (1990) pg 82.
 - [4] Kolb and Turner, *ibid* Eq 3.77.
 - [5] We note this does not depend on assuming a thermodynamic character for the energy density in question. If the particles in question are massive the shift to the present time is more complicated.
 - [6] Cosmic rays originating before recombination are a possibility in “top-down” models, see P. Bhattacharjee and G. Sigl, Physics Reports **327**, 109, (2000). We do not consider very high energy cosmic rays where effects of scattering on the CMB or infrared background become important.